



Pion loops in equal-time and light-front dynamics: *pseudovector vs. pseudoscalar theories*

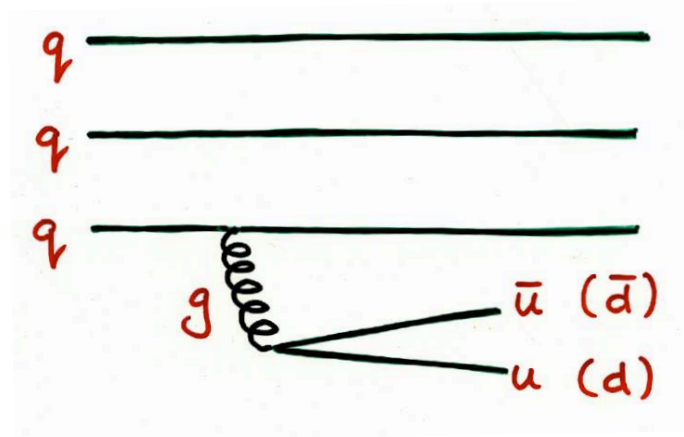
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with Chueng Ji, Khalida Hendricks (NCSU),
Tony Thomas (Adelaide)

Background

- Antiquarks in the proton “sea” produced predominantly by gluon radiation into quark-antiquark pairs, $g \rightarrow q\bar{q}$



→ since u and d quark masses are similar,
expect flavor-symmetric sea, $\bar{d} \approx \bar{u}$

- Experimentally, one finds *large excess* of \bar{d} over \bar{u}

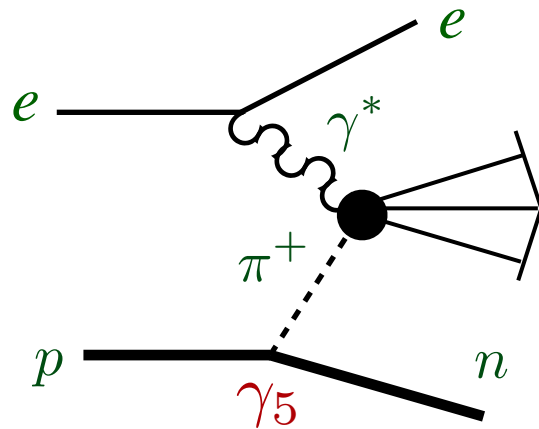
$$\int_0^1 dx (\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$

E866 (Fermilab), PRD 64, 052002 (2001)

Background

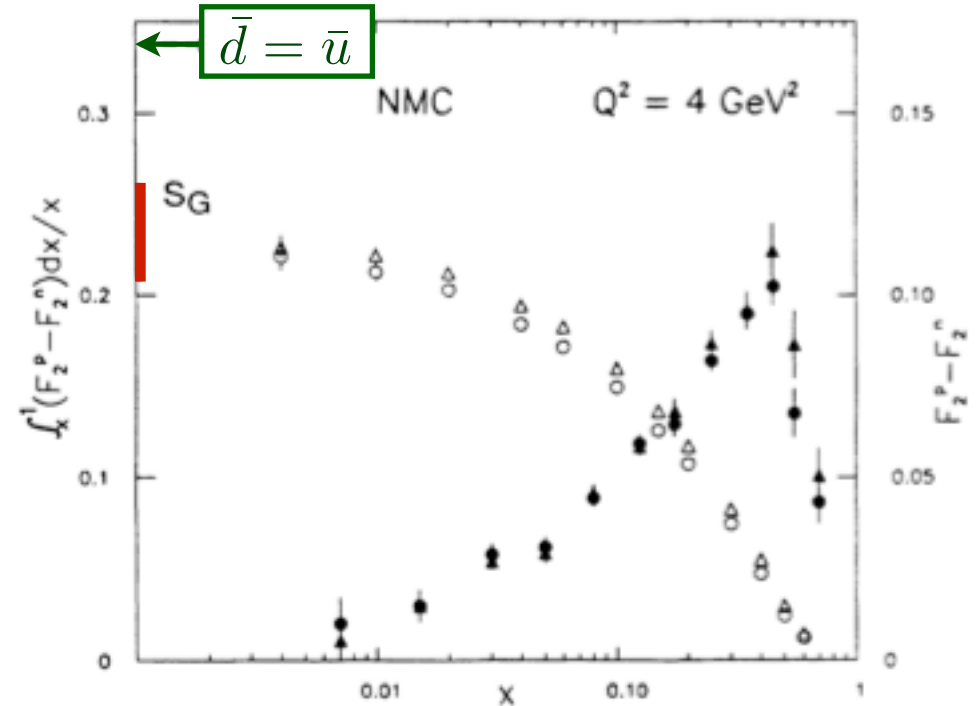
- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD 5, 1732 (1972)

Thomas, PLB 126, 97 (1983)



$$\int_0^1 \frac{dx}{x} (F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3} \int_0^1 dx (\bar{d} - \bar{u})$$

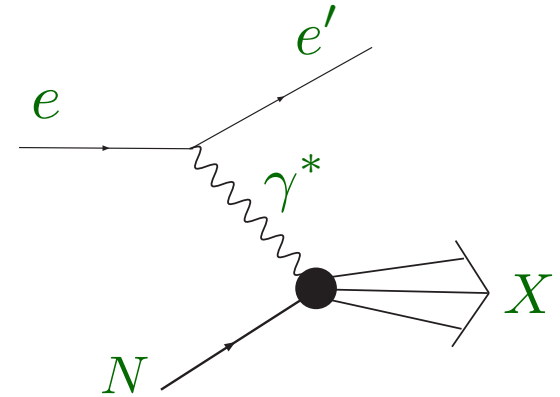
$$= 0.235(26)$$

New Muon Collaboration, PRD 50, 1 (1994)

Background

■ Inclusive cross section for $eN \rightarrow eX$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left(2 \tan^2 \frac{\theta}{2} \frac{F_1}{M} + \frac{F_2}{\nu} \right)$$



$$\left. \begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^2 - \nu^2 = 4EE' \sin^2 \frac{\theta}{2} \end{aligned} \right\} x = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

■ F_1, F_2 structure functions

→ contain all information about structure of nucleon

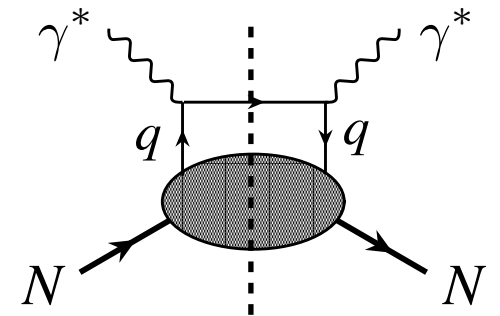
Background

- In *deep-inelastic region* ($W \gtrsim 2 \text{ GeV}$, $Q^2 \gtrsim 1 \text{ GeV}^2$)
structure function given by parton distribution functions

$$\begin{aligned} F_2(x, Q^2) &= x \sum_q e_q^2 q(x, Q^2) \\ &= \frac{4}{9}x(u + \bar{u}) + \frac{1}{9}x(d + \bar{d}) + \frac{1}{9}x(s + \bar{s}) + \dots \end{aligned}$$

→ $q(x, Q^2)$ = probability to find parton
type “ q ” in nucleon, carrying
momentum fraction x

“PDF”

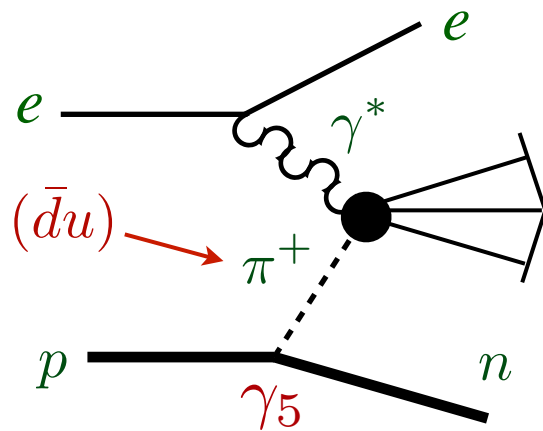


→ at large Q^2 , structure functions related by $F_2 = 2xF_1$

Background

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

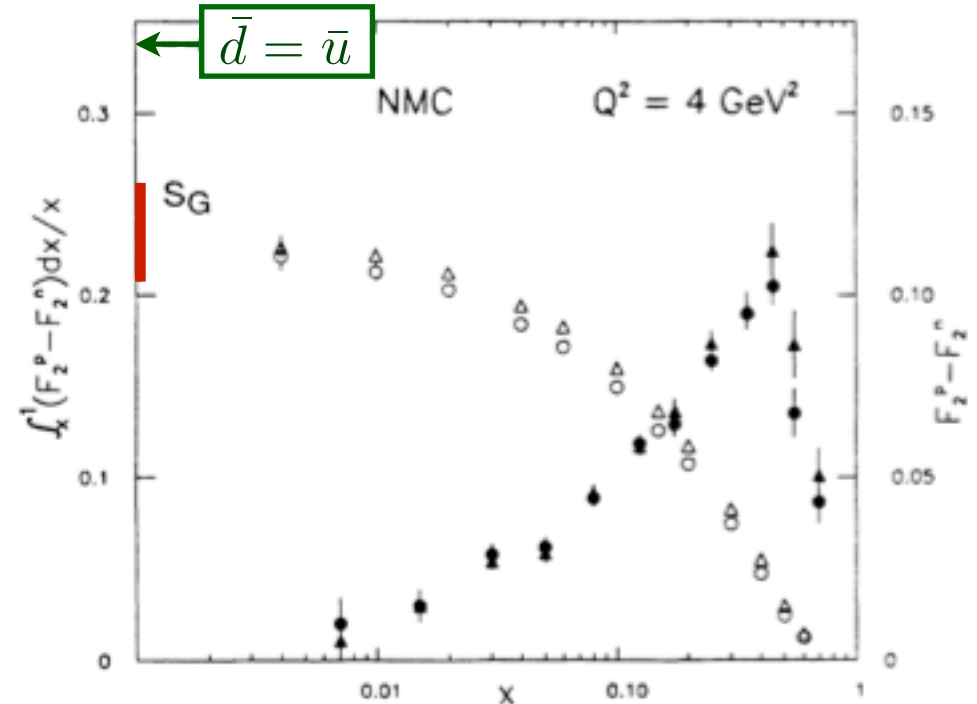
→ Sullivan process



Sullivan, PRD 5, 1732 (1972)

Thomas, PLB 126, 97 (1983)

$$\bar{d} > \bar{u}$$



$$\int_0^1 \frac{dx}{x} (F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3} \int_0^1 dx (\bar{d} - \bar{u})$$

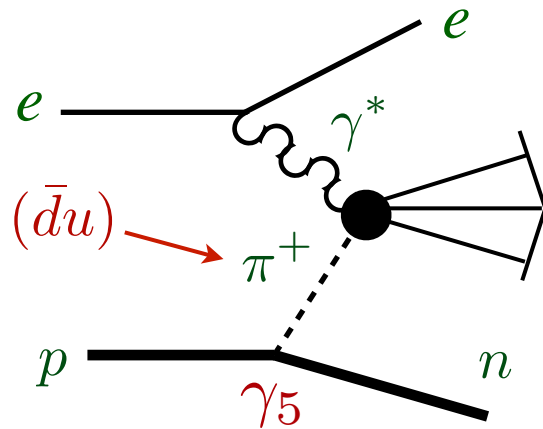
$$= 0.235(26)$$

New Muon Collaboration, PRD 50, 1 (1994)

Background

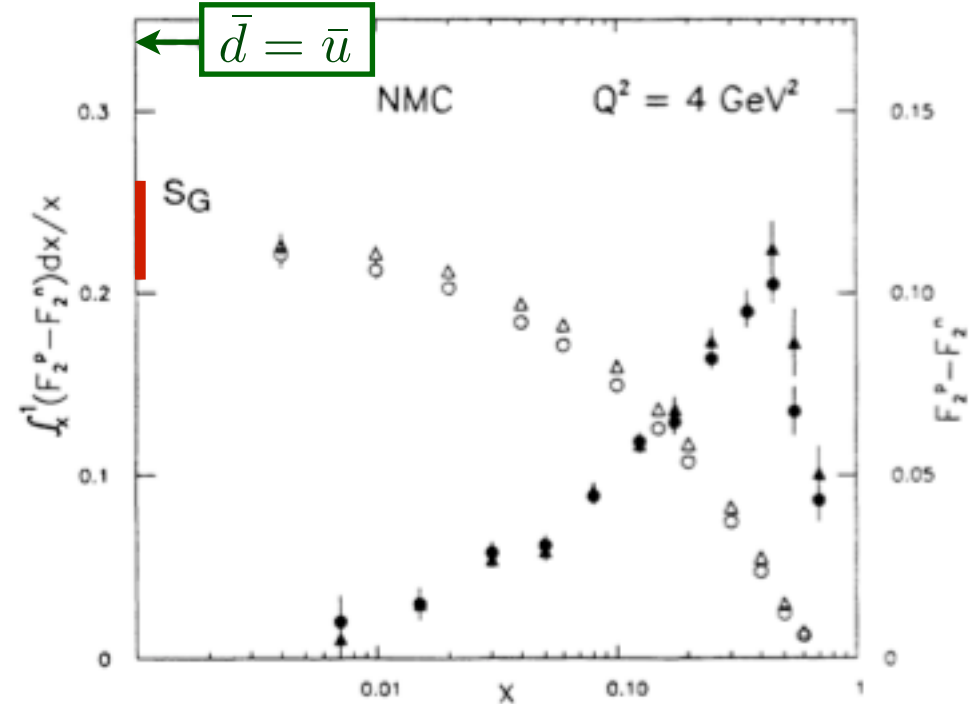
- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD 5, 1732 (1972)

Thomas, PLB 126, 97 (1983)



$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} f_\pi(y) \bar{q}^\pi(x/y)$$

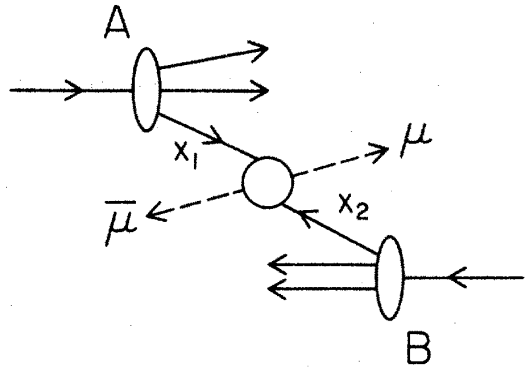
pion light-cone momentum distribution in nucleon

$$f_\pi(y) = \frac{3g_{\pi NN}^2}{16\pi^2} y \int dt \frac{-t \mathcal{F}_{\pi NN}^2(t)}{(t - m_\pi^2)^2}$$

Background

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

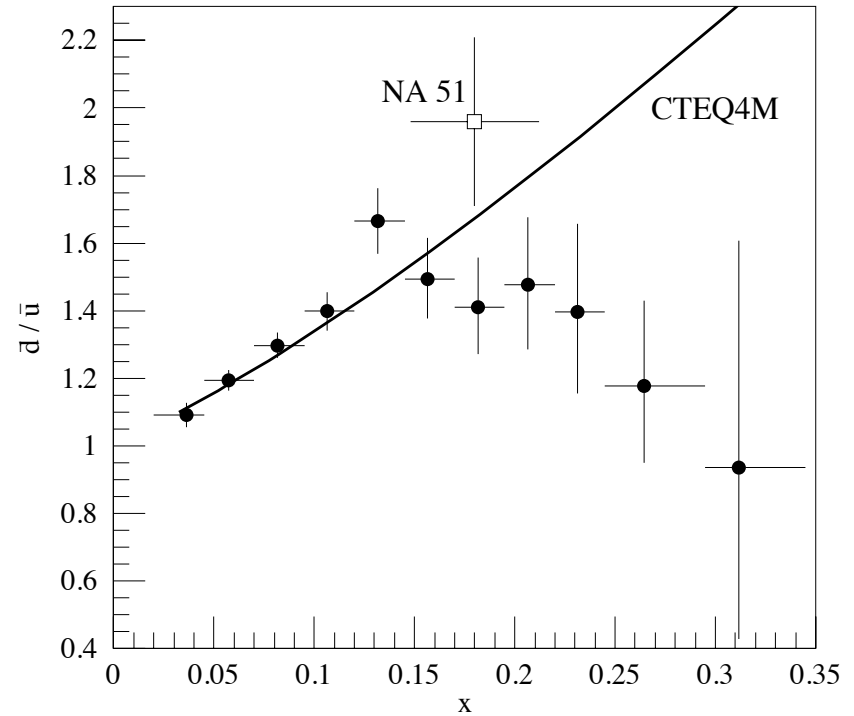
→ Drell-Yan process



$$\frac{d^2\sigma}{dx_b dx_t} = \frac{4\pi\alpha^2}{9Q^2} \sum_q e_q^2 (q(x_b)\bar{q}(x_t) + \bar{q}(x_b)q(x_t))$$

For $x_b \gg x_t$

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2} \left(1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)} \right)$$



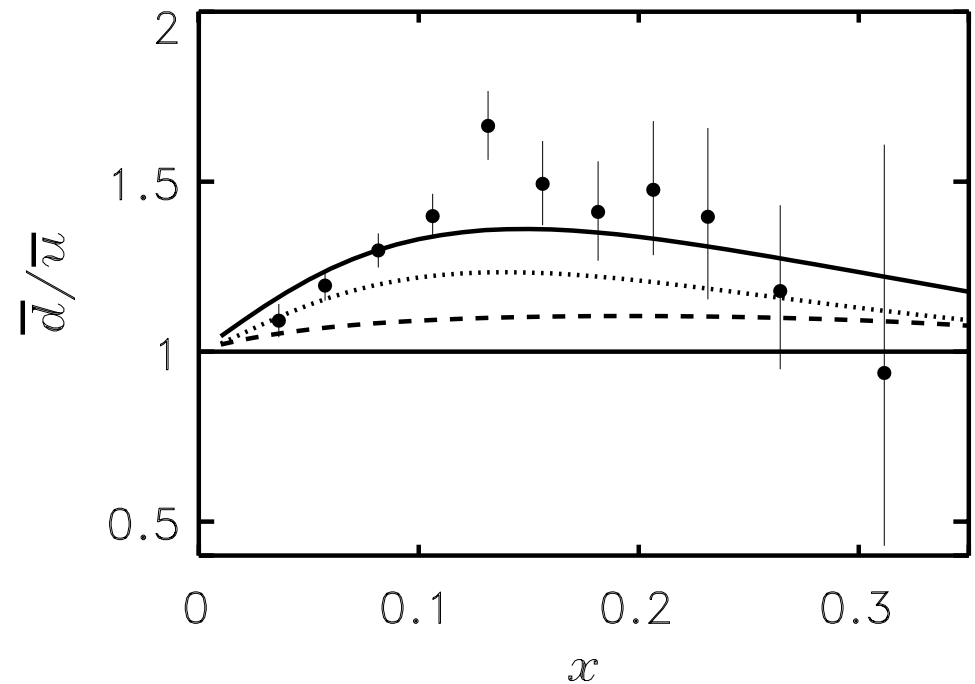
E866, PRL 80, 3715 (1998)

Background

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ reasonable description of data in model with π cloud + Pauli blocking

→ connection to QCD?



*WM, Speth, Thomas
PRD 59, 014033 (1998)*

Connection with QCD?

■ Chiral expansion of moments of $f_\pi(y)$

→ model-independent leading nonanalytic (LNA) behavior

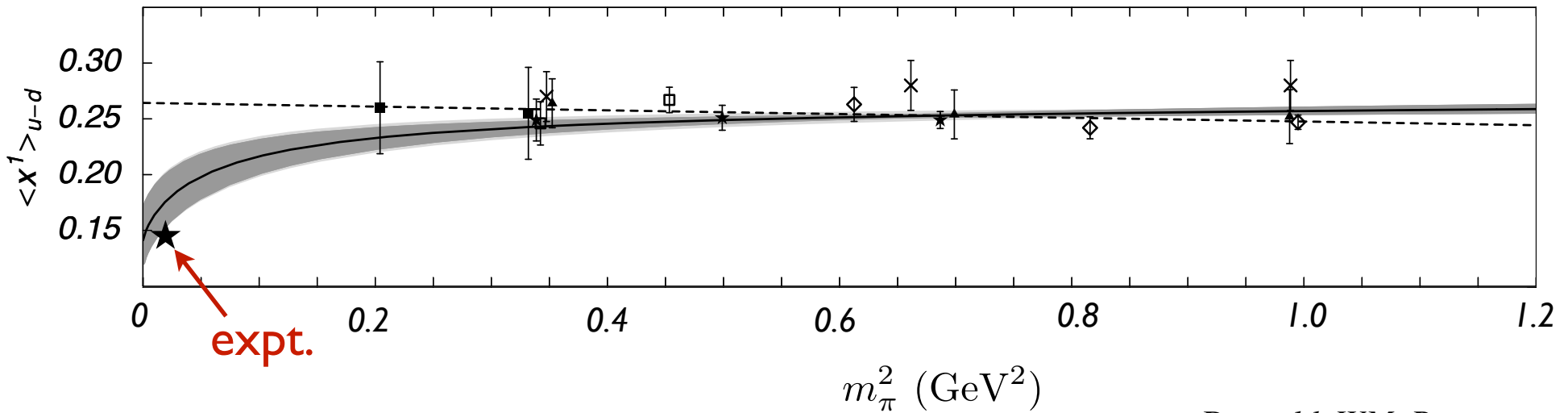
$$\begin{aligned}\langle x^0 \rangle_{\bar{d}-\bar{u}} &\equiv \int_0^1 dx (\bar{d} - \bar{u}) \\ &= \frac{2}{3} \int_0^1 dy f_\pi(y) \\ &= \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) + \text{analytic terms}\end{aligned}$$

*Thomas, WM, Steffens
PRL 85, 2892 (2000)*

→ can only be generated by pion cloud!

Connection with QCD?

- Nonanalytic behavior vital for chiral extrapolation of lattice data



*Detmold, WM, Renner et al.
PRL 87, 172001 (2001)*

→ allows lattice QCD calculations to be reconciled with experiment

Connection with QCD?

- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with “Sullivan” result

$$\langle x^n \rangle_{u-d} = a_n \left(1 + \frac{(3g_A^2 + 1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2 / \mu^2) \right) + \mathcal{O}(m_\pi^2)$$

cf. $4g_A^2$ in “Sullivan”, via moments of $f_\pi(y)$

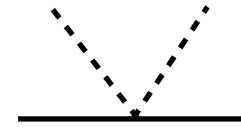
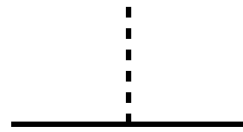
Chen, X. Ji, *PLB* **523**, 107 (2001)
Arndt, Savage, *NPA* **692**, 429 (2002)

- is there a problem with application of ChPT or “Sullivan process” to DIS?
- is light-front treatment of pion loops problematic?
- investigate relation between *covariant*, *instant-form*, and *light-front* formulations
- consider simple test case: nucleon self-energy

πN Lagrangian

■ Chiral (pseudovector) Lagrangian

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi} \bar{\psi}_N \gamma^\mu \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi} \psi_N - \frac{1}{(2f_\pi)^2} \bar{\psi}_N \gamma^\mu \vec{\tau} \cdot (\vec{\pi} \times \partial_\mu \vec{\pi}) \psi_N$$



$$g_A = 1.267$$
$$f_\pi = 93 \text{ MeV}$$

→ lowest order approximation of chiral perturbation theory Lagrangian

■ Pseudoscalar Lagrangian

$$\mathcal{L}_{\pi N}^{\text{PS}} = -g_{\pi NN} \bar{\psi}_N i\gamma_5 \vec{\tau} \cdot \vec{\pi} \psi_N$$

Self-energy

■ From lowest order PV Lagrangian

$$\Sigma = i \left(\frac{g_{\pi NN}}{2M} \right)^2 \bar{u}(p) \int \frac{d^4 k}{(2\pi)^4} (\not{k} \gamma_5 \vec{\tau}) \frac{i (\not{p} - \not{k} + M)}{D_N} (\gamma_5 \not{k} \vec{\tau}) \frac{i}{D_\pi^2} u(p)$$

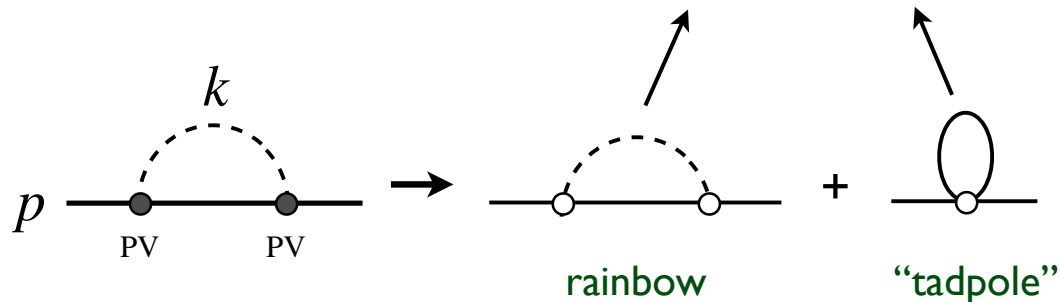
Goldberger-Treiman $\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M}$

$$D_\pi \equiv k^2 - m_\pi^2 + i\varepsilon$$

$$D_N \equiv (p - k)^2 - M^2 + i\varepsilon$$

→ rearrange in more transparent “reduced” form

$$\Sigma = -\frac{3ig_A^2}{4f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2M} \left[4M^2 \left(\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N} \right) + \frac{2p \cdot k}{D_\pi} \right]$$



C.-R. Ji, WM, Thomas, PRD **80**, 054018 (2009)

Self-energy

■ Covariant (dimensional regularization)

$$\int d^{4-2\varepsilon} k \frac{1}{D_\pi D_N} = -i\pi^2 \left(\gamma + \log \pi - \frac{1}{\varepsilon} + \int_0^1 dx \log \frac{(1-x)^2 M^2 + x m_\pi^2}{\mu^2} + \mathcal{O}(\varepsilon) \right)$$
$$\int d^{4-2\varepsilon} k \frac{1}{D_N} = -i\pi^2 M^2 \left(\gamma + \log \pi - \frac{1}{\varepsilon} + \log \frac{\mu^2}{M^2} + \mathcal{O}(\varepsilon) \right)$$

→ gives well-known m_π^3 LNA behavior
(from $1/D_\pi D_N$ term)

$$\Sigma_{\text{cov}}^{\text{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{1}{2\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

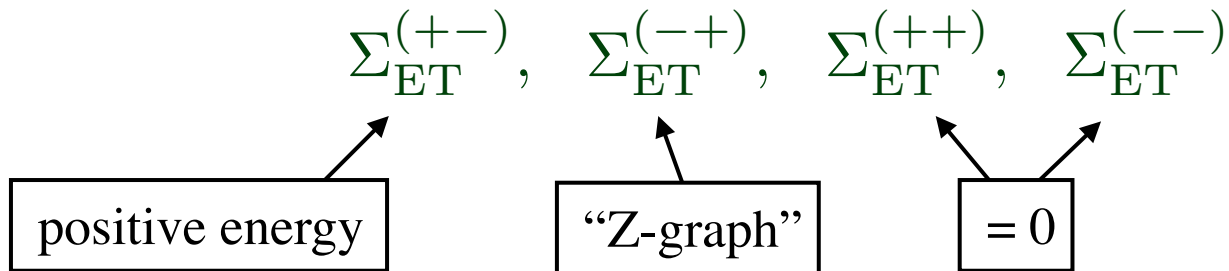
Self-energy

■ Equal time (rest frame)

$$\int d^4k \frac{1}{D_\pi D_N} = \int d^3k \int_{-\infty}^{\infty} dk_0 \frac{1}{(-2)(\omega_k - i\varepsilon)} \left(\frac{1}{k_0 - \omega_k + i\varepsilon} - \frac{1}{k_0 + \omega_k - i\varepsilon} \right) \\ \times \frac{1}{2(E' - i\varepsilon)} \left(\frac{1}{k_0 - E + E' - i\varepsilon} - \frac{1}{k_0 - E - E' + i\varepsilon} \right)$$

$$\omega_k = \sqrt{\mathbf{k}^2 + m_\pi^2}, \quad E' = \sqrt{\mathbf{k}^2 + M^2}$$

→ four time-orderings



$$\Sigma_{\text{ET}}^{(+-)\text{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{3}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

$$\Sigma_{\text{ET}}^{(-+)\text{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(-\frac{1}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

■ Equal time (infinite momentum frame)

$$\begin{aligned}\Sigma_{\text{IMF}}^{(+ -)} &= -\frac{3g_A^2 M}{16\pi^3 f_\pi^2} \int_{-\infty}^{\infty} dy \int d^2 k_\perp \frac{P}{2E'} \frac{1}{2\omega_k} \frac{m_\pi^2}{(E - E' - \omega_k)} \\ &= \frac{3g_A^2 M}{32\pi^2 f_\pi^2} \int_0^1 dy \int_0^{\Lambda^2} dk_\perp^2 \frac{m_\pi^2}{k_\perp^2 + M^2(1-y)^2 + m_\pi^2 y}\end{aligned}$$

$$p_z \equiv P \rightarrow \infty$$

$$y = p'_z/p_z$$

$$\Sigma_{\text{IMF}}^{(- +)} = \frac{3g_A^2 M}{16\pi^3 f_\pi^2} \int_{-\infty}^{\infty} dy \int d^2 k_\perp \frac{P}{2E'} \frac{1}{2\omega_k} \frac{m_\pi^2}{(E + E' + \omega_k)} = \mathcal{O}(1/P^2)$$

→ nonanalytic behavior same as in rest frame

$$\Sigma_{\text{IMF}}^{\text{LNA}} = \Sigma_{\text{IMF}}^{(+ -)\text{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{1}{2\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

■ Light-front

$$\begin{aligned}
 \int dk^+ dk^- d^2 k_\perp \frac{1}{D_\pi D_N} &= \frac{1}{p^+} \int_{-\infty}^{\infty} \frac{dx}{x(x-1)} d^2 k_\perp \int dk^- \left(k^- - \frac{k_\perp^2 + m_\pi^2}{xp^+} + \frac{i\varepsilon}{xp^+} \right)^{-1} \\
 &\quad \times \left(k^- - \frac{M^2}{p^+} - \frac{k_\perp^2 + M^2}{(x-1)p^+} + \frac{i\varepsilon}{(x-1)p^+} \right)^{-1} \\
 &= 2\pi^2 i \int_0^1 dx \, dk_\perp^2 \frac{1}{k_\perp^2 + (1-x)m_\pi^2 + x^2 M^2} \\
 &\qquad\qquad\qquad x = k^+/p^+
 \end{aligned}$$

→ identical nonanalytic results as covariant & instant form

$$\Sigma_{\text{LF}}^{\text{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{1}{2\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

■ Light-front

→ $1/D_N$ “tadpole” term has k^- pole that depends on k^+ and moves to infinity as $k^+ \rightarrow 0$

(“treacherous” in LF dynamics)

→ use LF cylindrical coordinates $k^+ = r \cos \phi$, $k^- = r \sin \phi$

$$\begin{aligned} \int d^4 k \frac{1}{D_N} &= \frac{1}{2} \int d^2 k_{\perp} \int \frac{dk^+}{k^+} \int dk^- \left(k^- - \frac{k_{\perp}^2 + M^2}{k^+} + \frac{i\varepsilon}{k^+} \right)^{-1} \\ &= -2\pi \int d^2 k_{\perp} \left[\int_0^{r_0} dr \frac{r}{\sqrt{r_0^4 - r^4}} + i \lim_{R \rightarrow \infty} \int_{r_0}^R dr \frac{r}{\sqrt{r^4 - r_0^4}} \right] \\ &= \frac{1}{2} \int d^2 k_{\perp} \lim_{R \rightarrow \infty} \left(-\pi^2 + 2\pi i \log \frac{r_0^2}{R^2} + \mathcal{O}(1/R^4) \right) \end{aligned}$$

relevant also
for $1/D_{\pi}$ term

contains $\log(k_{\perp}^2 + M^2)$
term as required

$$r_0 = \sqrt{2(k_{\perp}^2 + M^2)}$$

Self-energy

■ Pseudoscalar interaction

$$\begin{aligned}\Sigma^{\text{PS}} &= ig_{\pi NN}^2 \bar{u}(p) \int \frac{d^4 k}{(2\pi)^4} (\gamma_5 \vec{\tau}) \frac{i(\not{p} - \not{k} + M)}{D_N} (\gamma_5 \vec{\tau}) \frac{i}{D_\pi^2} u(p) \\ &= -\frac{3ig_A^2 M}{2f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \left[\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N} - \frac{1}{D_\pi} \right]\end{aligned}$$

- contains additional (“treacherous”) pion “tadpole” term
- similar evaluation as for $1/D_N$ term

$$\Sigma_{\text{LNA}}^{\text{PS}} = \frac{3g_A^2}{32\pi f_\pi^2} \left(\frac{M}{\pi} m_\pi^2 \log m_\pi^2 - m_\pi^3 - \frac{m_\pi^4}{2\pi M^2} \log \frac{m_\pi^2}{M^2} + \mathcal{O}(m_\pi^5) \right)$$

additional *lower order* term in PS theory!

Self-energy

- Alberg & Miller claim on light-front $\Sigma^{\text{PS}} = \Sigma^{\text{PV}}$
 - “form factor removes $k^+ = 0$ contribution” *PRL* **108**, 172001 (2012)

- In practice, AM drop “treacherous” $k^+ = 0$ (end-point) term

$$\Sigma^{\text{PS}} = \Sigma^{\text{PV}} + \Sigma_{\text{end-pt}}^{\text{PS}}$$

after which PS result happens to coincide with PV

→ but, *even with* form factors, end-point term is non-zero

$$\Sigma_{\text{end-pt}}^{\text{PS}} = \frac{3g_A^2 M}{16\pi^2 f_\pi^2} \int_0^\infty dt \frac{\sqrt{t} F^2(m_\pi^2, -t)}{\sqrt{t + m_\pi^2}} \xrightarrow{\text{LNA}} \frac{3g_A^2}{32\pi f_\pi^2} \frac{M}{\pi} m_\pi^2 \log m_\pi^2$$

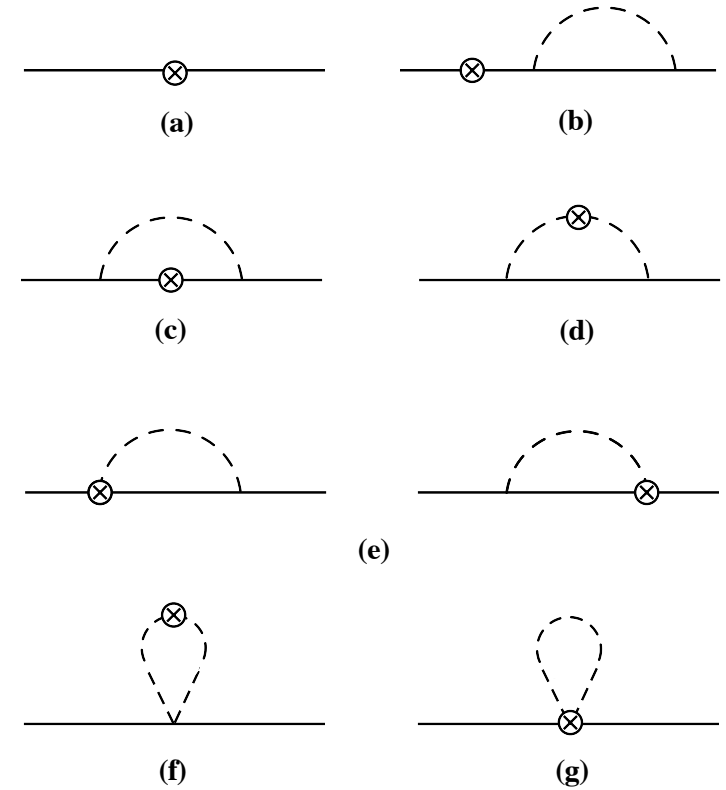
Ji, WM, Thomas, arXiv:1206.3671

→ *ansatz* does not work for other quantities
e.g. vertex renormalization

Vertex corrections

■ Pion cloud corrections to electromagnetic N coupling

→ N rainbow (c), π rainbow (d), Weinberg-Tomozawa (e), π tadpole (f), N tadpole (g)



■ Vertex renormalization

$$(Z_1^{-1} - 1) \bar{u}(p) \gamma^\mu u(p) = \bar{u}(p) \Lambda^\mu u(p)$$

→ taking “+” components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \frac{M}{p^+} \bar{u}(p) \Lambda^+ u(p)$

→ e.g. for N rainbow contribution,

$$\Lambda_\mu^N = -\frac{\partial \hat{\Sigma}}{\partial p^\mu}$$

Vertex corrections

■ Define light-cone momentum distributions $f_i(y)$

$$1 - Z_1^i = \int dy f_i(y)$$

for isovector (p - n) distribution

where

$$f_\pi(y) = 4f^{(\text{on})}(y) + 4f^{(\delta)}(y)$$

$$f_N(y) = -f^{(\text{on})}(y) - f^{(\text{off})}(y) + f^{(\delta)}(y)$$

$$f_{\text{WT}}(y) = 4f^{(\text{off})}(y) - 8f^{(\delta)}(y)$$

$$f_{\pi(\text{tad})}(y) = -f_{N(\text{tad})}(y) = 2f^{(\text{tad})}(y)$$

with components

$$f^{(\text{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}$$

$$f^{(\text{off})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}$$

$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$

$$f^{(\text{tad})}(y) = -\frac{1}{(4\pi f_\pi)^2} \int dk_\perp^2 \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$

Burkardt, Hendricks, Ji, WM,
Thomas, arXiv:1211.5853

- Pion distribution $f_\pi(y)$ contains *on-shell* contribution $f^{(\text{on})}(y)$ equivalent to PS (“Sullivan”) result
- Nucleon distribution $f_N(y)$ contains in addition new *off-shell* contribution $f^{(\text{off})}(y)$
- Both contain singular $\delta(y)$ components $f^{(\delta)}(y)$, which are present only in PV theory
- Weinberg-Tomozawa term $f^{(\text{WT})}(y)$ needed for gauge invariance

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\text{WT}})$$
- Nucleon and pion tadpole terms equal & opposite

$$(1 - Z_1^{\pi(\text{tad})}) + (1 - Z_1^{N(\text{tad})}) = 0$$

■ Nonanalytic behavior of vertex renormalization factors

$$1 - Z_1^N \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ m_\pi^2 \log m_\pi^2 - \pi \frac{m_\pi^3}{M} - \frac{2m_\pi^4}{3M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^\pi \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ m_\pi^2 \log m_\pi^2 - \frac{5\pi}{3} \frac{m_\pi^3}{M} - \frac{m_\pi^4}{M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{\text{WT}} \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ \frac{2\pi}{3} \frac{m_\pi^3}{M} - \frac{m_\pi^4}{3M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{N(\text{tad})} \xrightarrow{\text{NA}} -\frac{1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$1 - Z_1^{\pi(\text{tad})} \xrightarrow{\text{NA}} \frac{1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

→ cancellation of $m_\pi^2 \log m_\pi^2$ terms in WT contribution

→ demonstration of gauge invariance condition
(in fact, to *all* orders!)

■ Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1 - Z_1^N$	$g_A^2 *$	0	$-\frac{1}{2}g_A^2$	$\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^\pi$	0	$g_A^2 *$	0	$-\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^{\text{WT}}$	0	0	$-\frac{1}{2}g_A^2$	$\frac{1}{2}g_A^2$	0	0
$1 - Z_1^{N \text{ tad}}$	0	0	0	$-1/2$	$-1/2$	0
$1 - Z_1^{\pi \text{ tad}}$	0	0	0	$1/2$	$1/2$	0

* also in PS

in units of $\frac{1}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$

→ origin of ChPT vs. Sullivan process difference clear!

$$\left(1 - Z_1^{N \text{ (PV)}}\right)_{\text{LNA}} = \frac{3}{4} \left(1 - Z_1^{N \text{ (PS)}}\right)_{\text{LNA}}$$

Moments of PDFs

- PDF moments related to nucleon matrix elements of local twist-2 operators

$$\langle N | \hat{\mathcal{O}}_q^{\mu_1 \cdots \mu_n} | N \rangle = 2 \langle x^{n-1} \rangle_q p^{\{\mu_1 \cdots \mu_n\}}$$

→ n -th moment of (spin-averaged) PDF $q(x)$

$$\langle x^{n-1} \rangle_q = \int_0^1 dx x^{n-1} (q(x) + (-1)^n \bar{q}(x))$$

→ operator is $\hat{\mathcal{O}}_q^{\mu_1 \cdots \mu_n} = \bar{\psi} \gamma^{\{\mu_1} i D^{\mu_2} \cdots i D^{\mu_n\}} \psi$ – traces

- Lowest ($n=1$) moment $\langle x^0 \rangle_q \equiv \mathcal{M}_N + \mathcal{M}_\pi$ given by vertex renormalization factors $\sim 1 - Z_1^i$

Moments of PDFs

■ For couplings involving nucleons

$$\mathcal{M}_N^{(p)} = Z_2 + (1 - Z_1^N) + (1 - Z_1^{N(\text{tad})})$$

$$\mathcal{M}_N^{(n)} = 2(1 - Z_1^N) - (1 - Z_1^{N(\text{tad})})$$

→ wave function renormalization

$$1 - Z_2 = (1 - Z_1^p) + (1 - Z_1^n) \equiv 3(1 - Z_1^N)$$

■ For couplings involving only pions

$$\mathcal{M}_\pi^{(p)} = 2(1 - Z_1^\pi) + 2(1 - Z_1^{\text{WT}}) + (1 - Z_1^{\pi(\text{tad})})$$

$$\mathcal{M}_\pi^{(n)} = -2(1 - Z_1^\pi) - 2(1 - Z_1^{\text{WT}}) - (1 - Z_1^{\pi(\text{tad})})$$

■ Nonanalytic behavior

$$\mathcal{M}_N^{(p)} \xrightarrow{\text{LNA}} 1 - \frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p)} \xrightarrow{\text{LNA}} \frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_N^{(n)} \xrightarrow{\text{LNA}} \frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(n)} \xrightarrow{\text{LNA}} -\frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

→ no pion corrections to isosclar moments

→ isovector correction agrees with ChPT calculation

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{(4g_A^2 + [1 - g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{(4g_A^2 + [1 - g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

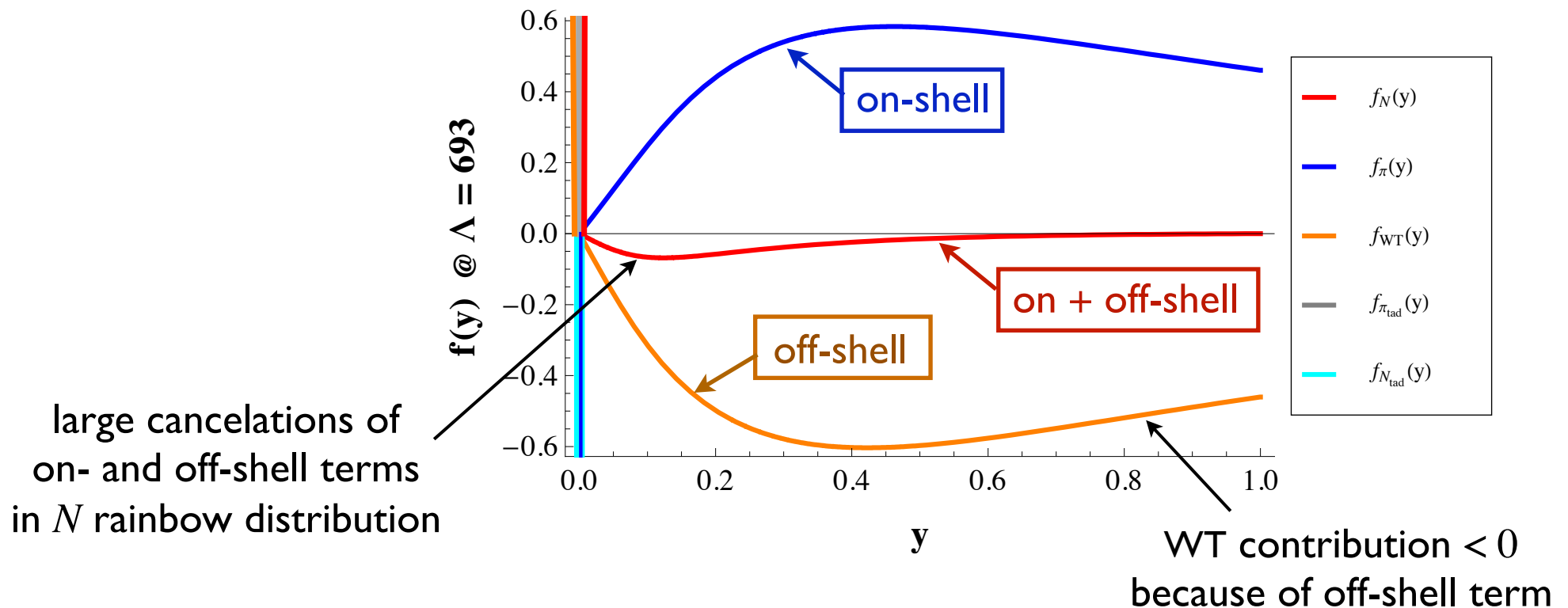
PS (“on-shell”) contribution

δ -function contribution

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

→ for transverse momentum cut-off $F(k_\perp) = \Theta(k_\perp^2 - \Lambda^2)$

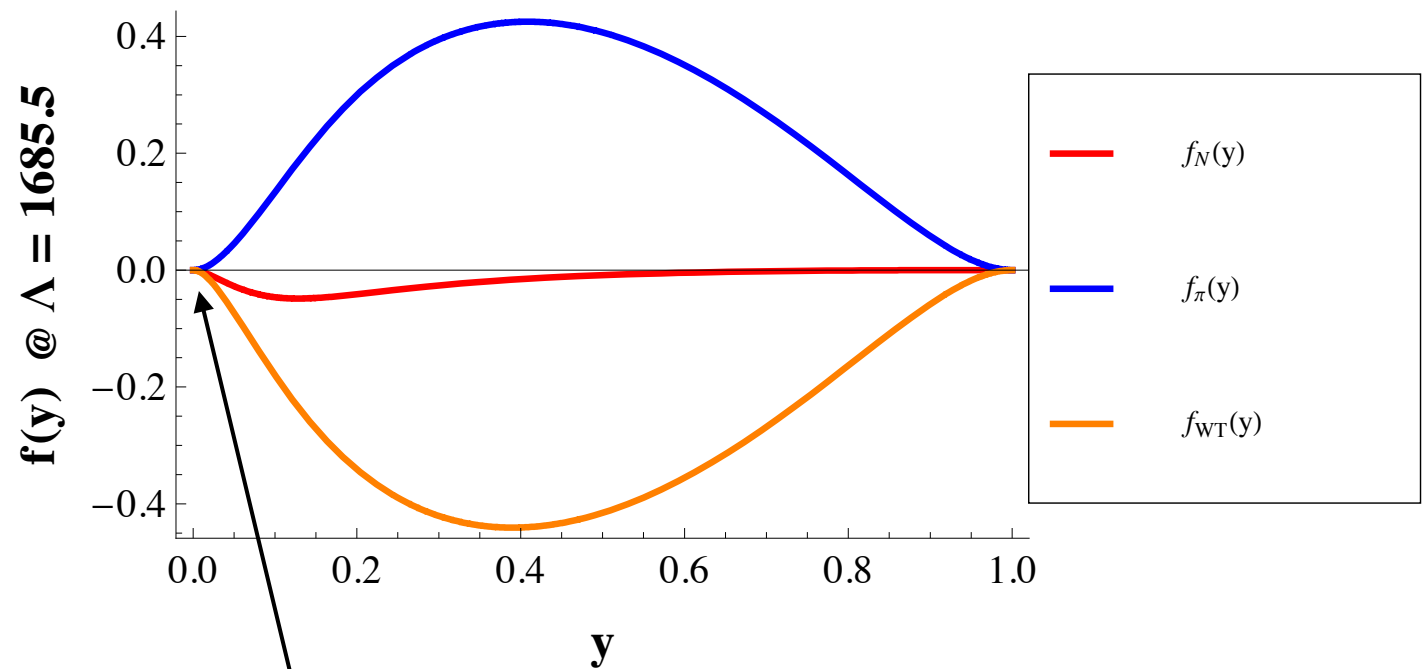


Hendricks, Ji, WM, Thomas (2013)

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

→ s -dependent (dipole) form factor $s_{\pi N} = \frac{k_{\perp}^2 + m_{\pi}^2}{y} + \frac{k_{\perp}^2 + M^2}{1-y}$



suppresses contributions
at $y=0$ and $y=1$
- no tadpoles!

Hendricks, Ji, WM, Thomas (2013)

Summary

- Equivalence demonstrated between self-energy in equal-time, covariant, and light-front formalisms

$$\Sigma_{\text{cov}}^{\text{LNA}} = \Sigma_{\text{ET}}^{(+ -)\text{LNA}} + \Sigma_{\text{ET}}^{(- +)\text{LNA}} = \Sigma_{\text{IMF}}^{(+ -)\text{LNA}} = \Sigma_{\text{LF}}^{\text{LNA}}$$

- non-trivial due to end-point singularities
- PV and PS results clearly differ

- Vertex corrections satisfy gauge invariance relations

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\text{WT}})$$

- difference between PDF moments in ChPT (PV) & “Sullivan” process (PS)
- model-independent constraints on pion light-cone momentum distributions
- impact on $\bar{d} - \bar{u}$ data analysis in progress